From Policy Gradient to Actor-Critic methods

The policy gradient derivation (3/3)

Olivier Sigaud

Sorbonne Université
http://people.isir.upmc.fr/sigaud

Policy Gradient with constant baseline

- Reminder:

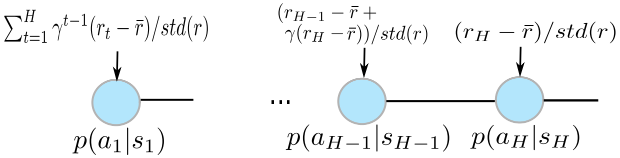
$$\nabla_{\boldsymbol{\theta}} J(\boldsymbol{\theta}) = \frac{1}{m} \sum_{i=1}^{m} \sum_{t=1}^{H} \nabla_{\boldsymbol{\theta}} \log \pi_{\boldsymbol{\theta}}(\mathbf{a}_t^{(i)}|\mathbf{s}_t^{(i)})[\sum_{k=t}^{H} \gamma^k r(\mathbf{s}_k^{(i)}, \mathbf{a}_k^{(i)})] \qquad (1)$$

- If all rewards are positive, the gradient increases all probabilities
- But with renormalization, only the largest increases emerge
- We can substract a "baseline" to (1) without changing its mean:

$$\nabla_{\boldsymbol{\theta}} J(\boldsymbol{\theta}) = \frac{1}{m} \sum_{i=1}^{m} \sum_{t=1}^{H} \nabla_{\boldsymbol{\theta}} \log \pi_{\boldsymbol{\theta}}(\mathbf{a}_t^{(i)}|\mathbf{s}_t^{(i)})[\sum_{k=t}^{H} \gamma^k r(\mathbf{s}_k^{(i)}, \mathbf{a}_k^{(i)}) - b]$$

- A first baseline is the average return $\bar{r}$ over all states of the batch
- Intuition: returns greater than average get positive, smaller get negative
- Use $(r_t^{(i)} - \bar{r})$ and divide by std $\rightarrow$ get a mean $= 0$ and a std $= 1$
- This improves variance (does the job of renormalization)
- Suggested in https://www.youtube.com/watch?v=tqrcjHuNdmQ

Algorithm 4: adding a constant baseline



- Estimate $\bar{r}$ and $std(r)$ from all rollouts
- Same as Algorithm 2, using $(r_t^{(i)} - \bar{r})/std(r)$
- Suffers from even less variance
- Does not work if all rewards r are identical (e.g. CartPole)

Policy Gradient with state-dependent baseline

- No impact on the gradient as long as the baseline does not depend on action
- A better baseline is $b(\mathbf{s}_t) = V^\pi(\mathbf{s}_t) = \mathbb{E}_\tau[r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + ... + \gamma^{H-t} r_H]$
- The expectation can be approximated from the batch of trajectories
- Thus we get

$$\nabla_{\boldsymbol{\theta}} J(\boldsymbol{\theta}) = \frac{1}{m} \sum_{i=1}^{m} \sum_{t=1}^{H} \nabla_{\boldsymbol{\theta}} \log \pi_{\boldsymbol{\theta}}(\mathbf{a}_t^{(i)}|\mathbf{s}_t^{(i)})[Q^{\pi_{\boldsymbol{\theta}}}(\mathbf{s}_t^{(i)}|\mathbf{a}_t^{(i)}) - V^{\pi_{\boldsymbol{\theta}}}(\mathbf{s}_t^{(i)})]$$

- $A^\pi(\mathbf{s}_t, \mathbf{a}_t) = Q^\pi(\mathbf{s}_t|\mathbf{a}_t) - V^\pi(\mathbf{s}_t)$ is the advantage function
- And we get

$$\nabla_{\boldsymbol{\theta}} J(\boldsymbol{\theta}) = \frac{1}{m} \sum_{i=1}^{m} \sum_{t=1}^{H} \nabla_{\boldsymbol{\theta}} \log \pi_{\boldsymbol{\theta}}(\mathbf{a}_t^{(i)}|\mathbf{s}_t^{(i)}) A^{\pi_{\boldsymbol{\theta}}}(\mathbf{s}_t^{(i)}, \mathbf{a}_t^{(i)})$$

https://www.youtube.com/watch?v=S_gwYj1Q-44 (27')

Williams, R. J. (1992) Simple statistical gradient-following algorithms for connectionist reinforcement learning. *Machine Learning*, 8(3-4):229–256

Estimating $V^\pi(s)$

- As for estimating $Q^\pi(s, a)$, but simpler
- Two approaches:
  - Monte Carlo estimate: Regression against empirical return

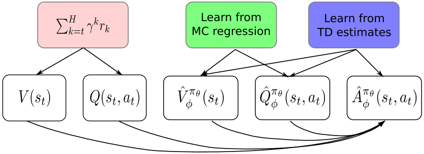
$$\boldsymbol{\phi}_{j+1} \rightarrow arg \min_{\boldsymbol{\phi}_j} \frac{1}{m} \sum_{i=1}^{m} \sum_{t=1}^{H} ((\sum_{k=t}^{H} r(\mathbf{s}_t^{(i)}, \mathbf{a}_t^{(i)})) - \hat{V}_{\boldsymbol{\phi}_j}^{\pi}(\mathbf{s}_t^{(i)}))^2$$

  - Temporal Difference estimate: init $\hat{V}_{\boldsymbol{\phi}_0}^{\pi}$ and fit using $(\mathbf{s}, \mathbf{a}, r, \mathbf{s}')$ data

$$\boldsymbol{\phi}_{j+1} \rightarrow \min_{\boldsymbol{\phi}_j} \sum_{(\mathbf{s}, \mathbf{a}, r, \mathbf{s}')} ||r + \gamma \hat{V}_{\boldsymbol{\phi}_j}^{\pi}(\mathbf{s}') - \hat{V}_{\boldsymbol{\phi}_j}^{\pi}(\mathbf{s})||^2$$

  - May need some regularization to prevent large steps in ϕ

Algorithm 5: adding a state-dependent baseline



- Learn $\hat{V}^{\pi}_{\boldsymbol{\phi}}$ from TD, from MC rollouts, or compute $V^{\pi_{\boldsymbol{\theta}}}(\mathbf{s}_t^{(i)})$ from MC
- Learn $\hat{Q}^{\pi}_{\boldsymbol{\phi}'}$ from TD, from MC rollouts, or compute $Q^{\pi_{\boldsymbol{\theta}}}(\mathbf{s}_t^{(i)}, \mathbf{a}_t^{(i)})$ from MC
- Compute $\hat{A}^{\pi}(\mathbf{s}_t^{(i)}|\mathbf{a}_t^{(i)}) = \hat{Q}^{\pi}_{\boldsymbol{\phi}}(\mathbf{s}_t^{(i)}, \mathbf{a}_t^{(i)}) - \hat{V}^{\pi}_{\boldsymbol{\phi}}(\mathbf{s}_t^{(i)})$
- Or even learn $\hat{A}^{\pi}_{\boldsymbol{\phi}}$ directly from TD updates using $A^{\pi}(\mathbf{s}_t, \mathbf{a}_t) = \mathbb{E}[\delta_t]$
- Same as Algorithm 3 using $A^{\pi_{\boldsymbol{\theta}}}(\mathbf{s}_t^{(i)}|\mathbf{a}_t^{(i)})$ instead of $Q^{\pi_{\boldsymbol{\theta}}}(\mathbf{s}_t^{(i)}|\mathbf{a}_t^{(i)})$
- Suffers from even less variance

Synthesis

No critic
- $\mathcal{R}(\tau)$
- $\sum_{k=t}^{H} r_k$
- $\sum_{k=t}^{H} \gamma^k r_k = Q(s_t, a_t)$

With critic
- δ_t
- $\hat{Q}_{\phi}^{\pi_\theta}(s_t, a_t)$
- $\hat{A}_{\phi}^{\pi_\theta}(s_t, a_t)$

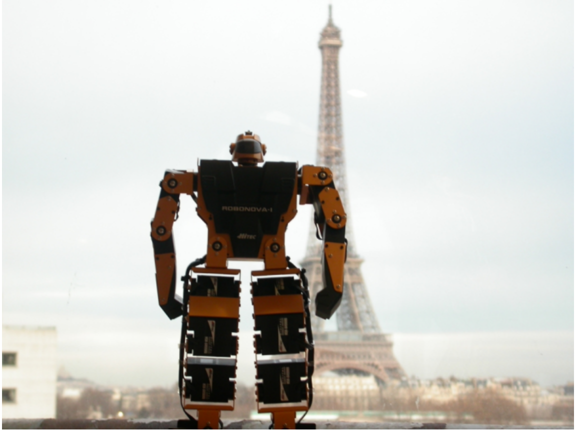
$\nabla_{\boldsymbol{\theta}} J(\boldsymbol{\theta}) = \mathbb{E}_{\mathbf{s}_t, \mathbf{a}_t \sim \pi_{\boldsymbol{\theta}}(.)}[\nabla_{\boldsymbol{\theta}} \log \pi_{\boldsymbol{\theta}}(\mathbf{a}_t^{(i)}|\mathbf{s}_t^{(i)})] \psi_t$ where ψ_t can be:

1. $\sum_{t=0}^{H} \gamma^t r_t$: total (discounted) reward of trajectory
2. $\sum_{k=t}^{H} \gamma^{k-t} r_k$: sum of rewards after $\mathbf{a}_t$
3. $\sum_{k=t}^{H} \gamma^{k-t} r_k - b(\mathbf{s}_t)$: sum of rewards after $\mathbf{a}_t$ with baseline
4. $\delta_t = r_t + \gamma V^\pi(\mathbf{s}_{t+1}) - V^\pi(\mathbf{s}_t)$: TD error, with $V^\pi(\mathbf{s}_t) = \mathbb{E}_{\mathbf{a}_t}[\sum_{k=0}^{H} \gamma^k r_{t+l}]$
5. $\hat{Q}_{\boldsymbol{\phi}}^{\pi_{\boldsymbol{\theta}}}(\mathbf{s}_t, \mathbf{a}_t) = \mathbb{E}_{a_{t+1}}[\sum_{k=0}^{H} \gamma^k r_{t+l}]$: action-value function
6. $\hat{A}_{\boldsymbol{\phi}}^{\pi_{\boldsymbol{\theta}}}(\mathbf{s}_t, \mathbf{a}_t) = \hat{Q}_{\boldsymbol{\phi}}^{\pi_{\boldsymbol{\theta}}}(\mathbf{s}_t, \mathbf{a}_t) - \hat{V}_{\boldsymbol{\phi}}^{\pi_{\boldsymbol{\theta}}}(\mathbf{s}_t) = \mathbb{E}[\delta_t]$, advantage function

- Next lesson: Difference to Actor-Critic

John Schulman, Philipp Moritz, Sergey Levine, Michael I. Jordan, and Pieter Abbeel. High-dimensional continuous control using generalized advantage estimation. *arXiv preprint arXiv:1506.02438*, 2015

Any question?



Send mail to: `Olivier.Sigaud@upmc.fr`

John Schulman, Philipp Moritz, Sergey Levine, Michael I. Jordan, and Pieter Abbeel.
High-dimensional continuous control using generalized advantage estimation.
arXiv preprint arXiv:1506.02438, 2015.

Ronald J. Williams.
Simple statistical gradient-following algorithms for connectionist reinforcement learning.
Machine Learning, 8(3-4):229–256, May 1992.
ISSN 0885-6125.
doi: 10.1007/BF00992696.